

### Section 1.7 (Combinations of Functions and Composite Functions)

- Before exploring combinations of functions, let's review the domain of a function
  - The domain of a function  $f(x)$  is the set of real numbers for which the value of  $f(x)$  is defined
  - In particular, we need to exclude numbers that cause **division by zero** in a rational function and numbers that result in the **root (even root) of a negative number**
    - Because  $x/0$  and  $\text{sqrt}(-1)$  are undefined (not real numbers)

Examples: Find the domain of each function (what values of  $x$  make this function work)

$$f(x) = x^2 + 3x - 17$$

$$g(x) = \frac{5x}{x^2 - 49}$$

$$h(x) = \sqrt{9x - 27}$$

- Given 2 functions, the sum, difference, product, and quotient of those functions are also functions
- The domain for these combined functions consist of all real numbers that are common to the domains of the 2 individual functions (except for sometimes the quotient)

Example: Research has shown that the equation  $A(x) = 2x - 15$  represents the number of games Alabama typically wins in a season and  $B(x) = x - 2$  represents the number of games Auburn wins ( $x$  represents the number of games played). How many games combined will Alabama and Auburn win if they play 10 games?

$$A(10) = \quad B(10) = \quad A(10) + B(10) = \quad (A + B)(10) =$$

- Given functions  $f$  and  $g$ , we have (the domain of these combinations consists of numbers that are in the domain of each of the functions)

$$\text{Sum: } (f + g)(x) = f(x) + g(x)$$

$$\text{Difference: } (f - g)(x) = f(x) - g(x)$$

$$\text{Product: } (fg)(x) = f(x) * g(x)$$

$$\text{Quotient: } \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Examples: If  $f(x) = x + 3$  and  $g(x) = 3x - 1$ , find the following (and the domain in each case)

$$(f + g)(x) =$$

$$(f - g)(x) =$$

$$(f * g)(x) =$$

$$(f/g)(x) =$$

- To find the composition of functions  $(f \circ g)(x) = f(g(x))$ , you basically plug the function  $g(x)$  into the function  $f(x)$  and vice versa

(composite functions are used when the output of one function is used as the input to another function as in applying discounts while shopping – see explanations on pp. 224-225 in the book)

- The domain of a composite function  $f(g(x))$  is the set of all numbers for  $x$  such that  $x$  is in the domain of  $g$  and  $g(x)$  is in the domain of  $f$

Example: At Hooper's store on Sesame Street, Virginia Tech jerseys are finally going on sale to make room for the shipment of the newly designed jerseys. Jerseys are already \$10 off, and you can take an additional 20% off of that reduced price. If you find a men's jersey originally marked at \$70 and a women's jersey originally marked at \$60, what would be the sale price of each jersey?

Example: If  $f(x) = 2x + 3$ , find  $f(\odot)$

Examples: If  $f(x) = x^2$  and  $g(x) = 2x + 1$ , find each composition

$$f(g(3)) =$$

$$g(f(3)) =$$

$$f(g(x)) = (f \circ g)(x) =$$

$$g(f(x)) =$$

Examples: If  $f(x) = \sqrt{x}$  and  $g(x) = x + 1$ , find  $(f \circ g)(x)$  and  $(g \circ f)(x)$

Example: Express the given function  $h$  as a composition of 2 functions  $f$  and  $g$  so that  $h(x) = f(g(x))$  where one of the functions is  $3x + 4$ .

$$h(x) = (3x + 4)^3$$

$$h(x) = \sqrt{3x + 4}$$

Look at online HW problems (especially the last couple) in class

Book problems: 3,5,9,17,19,35,39,43,45,57,61,63,65,71,73,75,77,91