

### Section 3.1 (Exponential Functions)

An **exponential function** with base  $b$  can be defined as  $f(x) = b^x$  where  $b$  is a positive constant other than 1 and  $x$  is any real number (notice the variable  $x$  IS the exponent in this case)

Examples include  $f(x) = 2^x$ ,  $g(x) = 10^x$ , and  $h(x) = \left(\frac{1}{2}\right)^{x-1}$

You can typically evaluate exponential expressions using calculator buttons ( $y^x$ ) or (^)

Example: It was recently discovered that at the Cookie Monster Mall in Sesame Street that the exponential function  $f(x) = 40(1.6)^x$  can be used to model spending. Here,  $f(x)$  is the average amount spent (in dollars) at the mall after  $x$  hours. Learn to use your calculator functions to find the average amount of money spent after 4 hours in the Mall.

$$f(4) = 40(1.6)^4 =$$

Exponential functions have a wide variety of applications, and a common base in exponential applications is the number **e**.

The number **e** is defined as the value that  $\left(1 + \frac{1}{n}\right)^n$  approaches as  $n$  gets bigger ( $n \rightarrow \infty$ ).

The irrational number **e**, approximately 2.72, is called the natural base, and should be available on your calculators as (**e**<sup>x</sup>).

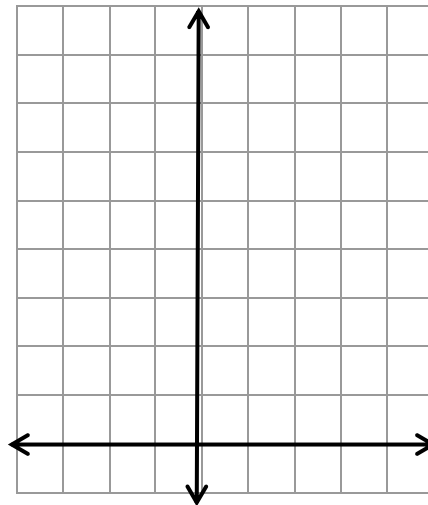
Example: Approximate  $e^{2.4}$  and  $e^{-0.67}$  using your calculator.

$$e^{2.4} = \qquad e^{-0.67} =$$

Let's look at some trends in the graphs of exponential functions

Example: Graph the following on the given axes by making a table of coordinates

$x$	$f(x) = 3^x$	$g(x) = \left(\frac{1}{3}\right)^x$
-2		
-1		
0		
1		
2		



**Discuss pages 391 and 392 for details on exponential graph characteristics**

Examples: Use transformations on pg. 392 to describe the graphs of the following...

$$f(x) = 3^{x+2} \qquad g(x) = e^x - 4 \qquad h(x) = (2)^{x-3} + 5p(x) = -e^x$$

f(x) is the graph of  $3^x$  shifted to the \_\_\_\_\_ by \_\_\_\_\_ places

g(x)

h(x)

p(x)

Examples: Graph one or more of the above examples or graph examples from the online HW

Money in savings accounts grows as a result of compound interest. Suppose you invest a sum of money (the principal) P at an annual percentage rate r compounded once per year (annually). Then the following table shows the accumulated value A of the money after so many years...

Time (years)	Value after compounding
0	$A = P$
1	$A = (P) + (P * r) = P(1 + r)$
2	$A = P(1 + r)^2$
3	$A = P(1 + r)^3$
t	$A = P(1 + r)^t$

Example: If you invested \$1000 today at a rate of 2% compounded annually, at the end of 2 years, your investment would be valued at  $A = 1000(1 + 0.02)^2 =$

Many savings accounts compound interest more than once a year. In general, when compound interest is paid n times per year (what is n if interest is compounded semi-annually? ), then in each time period, the interest rate is (r/n) and there are (nt) time periods in t years. Therefore after t years, the value of the investment is given by

$$A = P \left( 1 + \frac{r}{n} \right)^{nt} \quad \text{where } n \text{ is the number of compounding periods per year}$$

Furthermore, some banks use continuous compounding where the number of compounding periods increases infinitely ( $n \rightarrow \infty$ ). Looking at the effect (pg. 395), we see that for continuous compounding our investment value will be given by

$$A = Pe^{rt}$$

We can use these formulas to predict the value of investments

Example: Suppose Bill Gates has convinced you to invest that extra \$5000 you have lying around under your mattress. You want to invest the money for 4 years (while you get your garage constructed for your new Ferrari), and you have 2 options. You can invest in an account with a rate of 8% that compounds daily or an account with a rate of 7.75% that is compounded continuously. Which should you choose?