

Section 3.2 (Logarithmic Functions)

In the last section we defined an **exponential function** $f(x) = b^x$ where b is a positive constant other than 1 and x is any real number. Let's try to define the inverse of this function.

$$y = b^x \text{ (switch } x \text{ and } y) \quad \Rightarrow \quad x = b^y \text{ (solve for } y) \quad \Rightarrow \quad ???$$

The inverse function of the exponential function with base b is the **logarithmic function** with base b

$$f(x) = \log_b x \text{ is the logarithmic function with base } b \text{ where } y = \log_b x \text{ is equivalent to } b^y = x$$

There are 2 ways to express the same equation here

$\text{Logarithmic Form: } y = \log_b x$ <div style="display: flex; justify-content: center; gap: 20px;"> <div style="text-align: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">Exponent</div> <div style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">Base</div> </div> <div style="text-align: center;"> <div style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">Exponent</div> <div style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">Base</div> </div> </div>	$\text{Exponential Form: } b^y = x$
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Example: Write the following equations in equivalent exponential form

$3 = \log_7 x$

$\log_3 9 = y$

$3 = \log_b 64$

Example: Write the following equations in equivalent logarithmic form

$2^5 = x$

$b^3 = 27$

$\frac{1}{16} = 2^{-4}$

$\sqrt{9} = 3$

Given a logarithmic expression, you really ask yourself one question to evaluate the expression...

$\log_3 9 \text{ means } 3 \text{ to what power gives me } 9? \Rightarrow \text{ because } 3^2 = 9 \text{ then } \log_3 9 = 2$

$\log_{25} 5 \text{ means } 25 \text{ to what power gives me } 5? \Rightarrow \text{ because } 25^{??} = 5 \text{ then } \log_{25} 5 = \underline{\hspace{1cm}}$

Examples: Evaluate:

$\log_4 64$

$\log_5 \frac{1}{125}$

$\log_{36} 6$

$\log_3 \sqrt[5]{3}$

$\log_9 9$

$\log_8 1$

Re-examining the inverse of the exponential function ($f(x) = b^x$), we can now go a little further...

$$y = b^x \text{ (switch } x \text{ and } y) \quad \Rightarrow \quad x = b^y \text{ (solve for } y) \quad \Rightarrow \quad \mathbf{\log_b x = y}$$

This gives us the inverse properties of logarithms $\log_b b^x = x$ and $b^{\log_b x} = x$

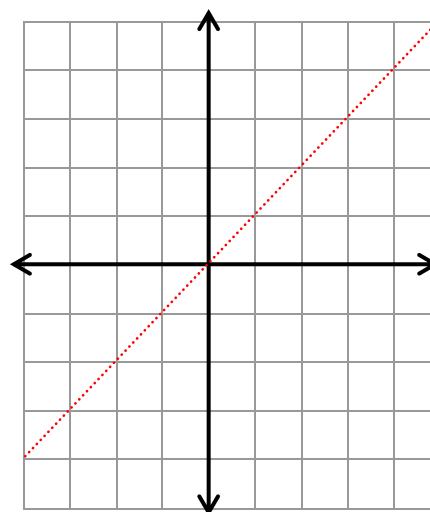
Examples: Evaluate $\log_7 7^8$ and $3^{\log_3 17}$

Considering that logarithmic functions are the inverses of exponential functions we know that their graphs should be symmetric about the line $y = x$...

Example: Graph $f(x) = 3^x$ and $g(x) = \log_3 x$ in the given coordinate system...

x	$f(x) = 3^x$
-2	
-1	
0	
1	
2	

x	$g(x) = \log_3 x$



Vertical Asymptote =>

REVIEW CHARACTERISTICS OF GRAPHS (pgs. 404-405)

Table 3.4 on pg. 405 discusses transformations of these logarithmic graphs

Examples: Use these properties to describe the graphs of $f(x) = \log_5(x + 3)$ and $g(x) = 5 - \log_2 x$

REVIEW ONLINE HW PROBLEMS 8-10

In looking at the domain of logarithmic functions, consider that in $y = \log_b x$ that we said b is a positive constant. So, if a positive number is raised to any power, the sign of the result is _____.

The domain of a logarithmic function of the form $f(x) = \log_b x$ is the set of all positive real numbers

Example: Find the domain of $f(x) = \log_4(x - 5)$

The logarithmic function with base 10 is called the *common logarithmic function* and the function $f(x) = \log_{10} x$ is usually expressed simply as $f(x) = \log x$ (the LOG button on calculators refers to the common log or $\log_{10} x$).

Example: Consider some properties of the common log (pg. 407) and use these properties to find $\log 10^6$

Furthermore, the logarithmic function with base e is called the *natural logarithmic function* and is usually expressed as $f(x) = \ln x$ (see pgs. 408-410 for properties and note the inverse properties).

Examples: Review online HW problems 13-14.